ISSN: 2278-4012, Volume:7, Issue:3, July 2018

MATHEMATICAL PROBLEM SOLVING HEURISTICS AND SOLUTION STRATEGIES OF SENIOR HIGH SCHOOL STUDENTS

Denis Abao Tan

Faculty, College of Education, Central Mindanao University,
University Town, Musuan, Bukidnon, 8710 Philippines

Abstract

Numerous researches from abroad had been investigating the mathematical problem solving process among students, but little is done with regards to the mathematical problem solving heuristics among high school students in the Philippines. This paper is a portion of a grounded theory research conducted by the researcher. This article presents the portion in which case study design was utilized to find out the mathematical problem solving heuristics among Filipino high school students. Thirty participants representing different problem solving skills ranging from poor to good problem solvers were interviewed over a period of three months. Interviews were analyzed and mathematical problem solving heuristics were identified. Filipino senior high school students utilized the following heuristics when they solved mathematical problems: use of logic or common sense, look for a clue, use an equation, draw a picture, solve or recall similar or equivalent problem, make a list, find a pattern, and make a table, make a diagram, use of mathematical expressions, use a model and use a representation. Other solution strategies used by the participants in the study were trial-and-error, guessing, guess-and-test, skip counting, use of the basic operations, use of adding techniques and recall and create formula. Participants in the study used the solution strategies as stand-alone or a combination of two or more strategies.

Keywords**:** *Problem Solving Heuristics, Mathematical Problem Solving, Senior Students*

INTRODUCTION

Problem solving has long been recognized as one of the hallmarks of mathematics. One of the greatest goals of mathematics education is to have students become good problem solvers (Billstein, Libeskind & Lott, 2000). Mathematics educators recognize the need to develop critical and analytic thinking through problem solving (Limjap & Candelaria, 2002). The 1989 Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) reflected a clear vision of the focal place of problem solving in Mathematics. Also, the 2000 Principles and Standards for School Mathematics reiterate the central place of problem solving at all levels of mathematics as follows:

"By learning problem solving in mathematics, students should acquire the ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages (NCTM, 2000, p. 52)".

According to Krulik and Rudnick (1996), problem solving is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. It begins with the initial confrontation and ends when an answer has been obtained and checked against the condition of the problems. They advocated that problem solving can and should be taught. The process has been analyzed and can be represented as a series of steps, referred to as a *heuristic plan*, or simply, *heuristics*. Heuristics provide a "road map"; a blueprint that directs one's path toward a solution and resolution of a problem situation. Although heuristics of mathematical problem solving has been identified, this was done abroad and not in the Philippine setting. This urges the researcher to conduct this study with a hope to identify problem solving heuristics of Filipino high school students specifically in Central Mindanao University, Bukidnon. This study is guided by Yin's (1989) case study approach.

This study aimed at understanding the problem solving heuristics of senior high school students in Central Mindanao University Laboratory High School (CMULHS).

METHODOLOGY

This section presents the set of procedures and techniques that were employed in gathering and analyzing the data in the study. It includes the research design, participants, data sources, instrument, procedures and data analysis that were utilized in this study.

Research Design

This study is mainly descriptive and qualitative in nature. Methods of case analysis were applied. The main purpose of this in-depth study is to examine the problem solving heuristics of the senior high school students of CMULHS.

Participants

The participants of this study were selected from the senior students of Central Mindanao University High School (CMULHS). The CMULHS populace is composed of 500 students (more or less) coming from different elementary schools in the nearby barangay and towns with more or less 100 seniors. The selection was made based on the results of the problem solving examination taken by the all the senior high school students prior to the conduct of the study.

RESULTS AND DISCUSSIONS

The mathematical problem solving heuristics and solution strategies presented below were used by the respondents in the different mathematical problems they solved. They claimed

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

that the use of a specific solution strategy depends on the problem type. Likewise, computational strategy depends on the degree of difficulty of the problem and the numbers involved. Most of the time students do not know the specific name of a strategy they used in solving a specific problem, but their actual solution of the problem indicates otherwise. They had learned and mastered few solution strategies in solving but they simply do not know them by their names. Some of the solution strategies and heuristics enumerated here were not mentioned by the respondents during the interview but were evident in their actual problem solving tasks as mentioned earlier. So, the researcher scanned their outputs for presentation and data triangulation purposes.

1. ***Trial-and-error.***

Seniors resort to this solution strategy when they are not familiar with the problem at hand. Most of the times, below average students use this strategy in solving mathematical problems. This was claimed by one of the below average respondents during the interview.

Researcher: "Unya unsa nimo pagdetermine kung unsa nga strategy imong gagamiton?" ("How will you determine the strategy you will use?")

Interviewee: "Maura ra man akong ginagamit, Ma'am, Trial-and-Error." ("I only used trial-and-error, Ma'am.")

The researcher observed that respondents who claimed that they did not like Mathematics resorted to trial-and-error strategy even without understanding the problem. They usually tend to use the four fundamental operations in Math and manipulate all the data in the problem, then stop when they felt that they had arrived at a reasonable answer based on their own estimation of the answer. This was the answer of one of the respondents during the interview.

Researcher: "Unsa man imong gabuhaton sa data?" ("How will you treat the data?")

Interviewee: "Ibaylo-baylo ang data, Ma'am, ug gamitan ug lain-lain nga operation." ("I manipulate the data, Ma'am, using the different mathematical operations.")

2. ***Guessing.***

This strategy is used by all the groups in the study but with varying frequency. Mostly, above average students resorted to guessing when their other strategies failed to help them in arriving at the correct answer. They tend to guess the answer when they were running out of time

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

and did not want to leave any items unanswered. Also, average students did the same things as the above average students do. Their only difference is that, average students often guess than the above average ones.

But among others, guessing is prominent to below average students. They used this strategy because they did not know what to do in the first place. They did not have the complete grasp of what the problem is all about and how to do about it.

Moreover, most of the students did not know that they are using strategy(ies) in solving. They were not familiar with the word "strategy" in solving. This was evident in the following transcript:

Researcher: "Unsa man na mga strategy imong gagamiton in solving?"("What are the strategies you use in solving?")
Interviewee: "Kanang unsa Ma'am?" ("What do you mean, Ma'am?")
Researcher: "Kanang example lugar, ga trial and error ba ka, ga guessing ba ka, ga model ba ka, ga-unsa man imong buhaton? Or sa formula ra gyud ka ga-base or unsa man na mga strategy imong gagamiton?" ("Say for example, do you use trial-and-error, guessing, modeling or what you usually use in solving? Or you usually recall or create formula in solving, or any strategy that you frequently use.")
Interviewee: "Kon dili siya familiar Ma'am kay……" ("If it (referring to the problem) is not familiar then….")
Researcher: "Unsa man." ("Then what?")
Interviewee: "Guessing."

The above transcript exposed the very reason why students resorted to guessing. As stated above by the interviewee, he guesses when he is not familiar with the problem at hand. Then a follow-up question was asked by the researcher which gave way to the next strategy that he used in solving mathematical problems.

Researcher: Ah, guessing kon dili familiar? Kon familiar?("Ah, so you guess when the problem is not familiar? How about if it is familiar?")
Interviewee: "Kuan mag-analize, mag-apply ug formula." ("Well, I will analyze, then apply a formula.")

The interviewee confirmed that he used guessing strategy in unfamiliar problems, otherwise used the recalled or created formula in solving.

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

3. Look/Create/Recall for a Formula.

The transcript above confirmed that students recall/create or look for a formula in solving familiar mathematical problems. Others claimed that recalling or creating a formula was their main strategy in solving. This is supported by the interview transcript below.

Researcher: "Naa ba kay lahi-lahi nga strategy in solving varied math problems?"("Do you have different strategies in solving varied math problems?")

Interviewee: "Strategies….ah, usually kay ga rely man ko sa formula Ma'am dana na topic. Mao ra naa akong gina-apply." ("Strategies…ah, usually I rely on the formula in that specific topic, Ma'am. That's what I usually apply.")

The interviewee emphasized the use of formula discussed in a specific topic in solving. He added that teachers never gave examination or problem to be solved that was never discussed in class. So, all he had to do is recall and do something about the formula in that topic to solve the problems given by the teacher. This was the very reason why he frequently used formula in solving mathematical problems.

A follow-up question was asked by the researcher which gave way to a new strategy "Use of common sense or logic".

4. Use of Logic/Common Sense.

As stressed by the interviewee, this strategy is used when problems do not require formula in solving them. All you have to do is analyze the problem, use logic and your common sense in solving.

Researcher: "Kung unsa nga formula….ah… So wala kay lain strategy but kato rang formula ana nga topic imong gamiton. How about in-ani nga problem kay wala may formula?" ("So, you use the formula in a specific topic…..don't you have any other strategy in solving besides using the formula in a topic? How about those problems that do not require formula, like this one (pointing on a logic problem)?")

Interviewee: "Common sense…he5…. Naa man koy mga strategies Ma'am pero dili nako ma-explain…he5…."("Common sense…heheheh. I have strategies, Ma'am, in solving but I can't explain…heheheh.").

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

It was during this portion that the interviewee admitted he has another strategy used in solving mathematical problem only that he cannot find the right words to explain it. So, the researcher let him described the strategy he used. This gave way to another strategy in solving which is "look for a clue".

5. Look for a Clue.

It is a fact that teachers gave problem solving items during examinations that are connected to each other or even related. One of the respondents claimed that he "looks for clue(s)" in problems given in the test to solve for a problem therein. Below was the portion of the interview transcripts.

> *Researcher: "Unsa man. Unsa…..i-describe lang." ("What is that? What…you may describe it.")*
> *Interviewee: "Kanang mangita ko ug answer or hint sa problem sa lain nga mga items sa problem….sa problem solving pud ko mangita ug formula nga magamit nako ug solve sa other item." ("Well, I will look for hints, clues or even answers of the problem in other problems given…..I will find formula that I can use in solving the problem in other items in the problem solving part of the test.")*
> *Researcher: "Dawbi kung dili sila related?" ("What if they (problems) are not related?")*
> *Interviewee: "Wala nako'y choice, Ma'am pro ga try jud ko ug solve." ("I do not have any choice, Ma'am, but I do try my best to solve them (problems).")*

The last statement of the interviewee gave us an idea about his desire to solve mathematical problems given by the teacher. He has different strategies in solving which are dependent on the problem to be solved at hand.

6. Use an equation and draw a picture.

One of the strategies that the respondents used in solving is "use an equation" or "draw a picture" or both. The transcript below will give us a clearer view on when and why students use this strategy solely or in pair.

> *Researcher: "Unsa kasagara imong gagamiton in solving." ("What do you usually use in solving?")*
> *Interviewee: Equation, Ma'am.*
> *Researcher: "Ah, so kasagara ga make ka ug equation?" ("Ah, so usually you make use of equation?")*
> *Interviewee: Yes, Ma'am.*

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

Researcher: "So, wala gyu'y instance nga ga draw ka, gahimo ug graph……. ana?" ("Are there instances that you draw or make graph……or any?")
Interviewee: "Hehe….. pag Geometry Ma'am ga draw ko." ("Hehe…in Geometry, Ma'am, I usually draw.")
Researcher: "Sa other subjects,, unsa man imo gagamiton?" ("In other math subjects, what do you usually use?")
Interviewee: "Equation, in Calculus ug Trigo. Ay, drawing ug equations diay pag Trigo, Ma'am, especially katong sa kuan, sa solutions of right triangles especially sa application?" ("I use equation in Calculus and Trigo. Uhmm, I mean, I use drawing and equations, Ma'am, in Trigonometry especially in topics like solutions of right triangles and in the application.")

Seniors have different strategies in different mathematical problems with regards to branches of mathematics. This implies that strategy is indeed dependent on the problem at hand as it has been emphasized by the respondents.

7. Solve or Recall a similar or an equivalent problem.

Frequently this heuristics is used by beginner or dependent problem solver. They just recall sample problems solved by the teacher in class or examples of problems solved in the book they have read. One of their many reasons in choosing this heuristics is its availability and easy use. Let us consider the response of the respondent below.

Researcher: "Unsa man kaha imo usually gamiton, katong sa gamay pa ka hantud karon. Unsa man kaha kasagara imo gabuhaton kung magsolve ka ug problem?" ("What is the usual thing you do, when you were young until now? What do you usually do when solving a problem?")
Interviewee: "Ay, murag kuan Ma'am… kon unsa sa maestro." ("Oh…..i think…Ma'am…I follow what the teacher does.")
Researcher: "Ah… kon unsa sa maestro mao ra pud imong way?"("Uhm…you follow your teacher's way?")
Interviewee: "O." ("Yes.")
Researcher: "Dili saka ug try ug pangitag ug other way?"("Haven't you tried finding another way?")
Interviewee: "Usahay ra."("Sometimes.")
Researcher: "Kanus-a man na?" ("When was that?")
Interviewee: "Kon sayon ra ang problem kay murag naa siya formula na mas dali." ("If problem is difficult, I think there is an easy formula to be used.")

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

The respondent claimed that he followed the procedure that the teacher had introduced in the class. He believed that there is an easy formula to be used in a specific problem whenever he finds another way of solving a problem.

8. Use a model

This heuristics was being described by one of the respondents during the interview. She had observed that her classmate was doing this during test. Below is the portion of the transcripts that shows her revelation.

Researcher: "Tapos gagamit ba ka ug tables or graph? or magdraw ba kaha karon."(Well, do you used tables or graph? Do you even draw?")
Interviewee: "Depende, kay usahay kay kon di makaya ug kanang bana-bana lang bitaw na imagine. Kanang imagine-imagine lang."("It depends, because sometimes if I can't solve it through estimation using my imagination. Just simply imagining.")
Researcher: "Ah…".
Interviewee: "Kato bitaw'ng fold-fold? Ifold daw, i-fold na sad, subra kayo ra kay akong classmate kay nagkuha gyud ug papel unya ilang gi-fold fold. Ako wala."("That folding problem? It is folded, then folded again. My classmates were so OA, because they really got some paper and fold it. But I don't.")
Researcher: "So, modeling ilaha? Modeling man tawag ana. Unsa pa imo gabuhaton?" ("So, they are using a model. Modelling is the term for that. So, what else?")
Interviewee: "Kanang idrawing, kon imagine man gud…okey ra mag-imagine kon gagmay lang bitaw na numbers pero pagdaghan na kay…"("Well, I will draw, imagine…but I only imagine if numbers are small, but for larger or bigger numbers, I will…..")
Researcher: "ah….unsa lugar?"(Ah…you will what…")
Interviewee: "Ah…. (ha,ha,ha)…. Magsulat nalang."("Ah, (giggling)..I will just simply write.")

The above transcripts do not only reveal the used of model by the respondent's classmate but was simply describing her computational strategy. She used her imagination and mentally computes the problem when numbers are smaller but attested that she preferred writing them down when it involved larger or bigger numbers.

This shows that there are two computational strategies that were used by the seniors depending on the problem difficulty and the numbers given. These are mental and written computational strategies.

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

The heuristics that follow were not mentioned by the respondents during the interview but were used in their actual problem solving task. Scanned copies of the solved problem were pasted here for proof of use.

9. Use a representation.

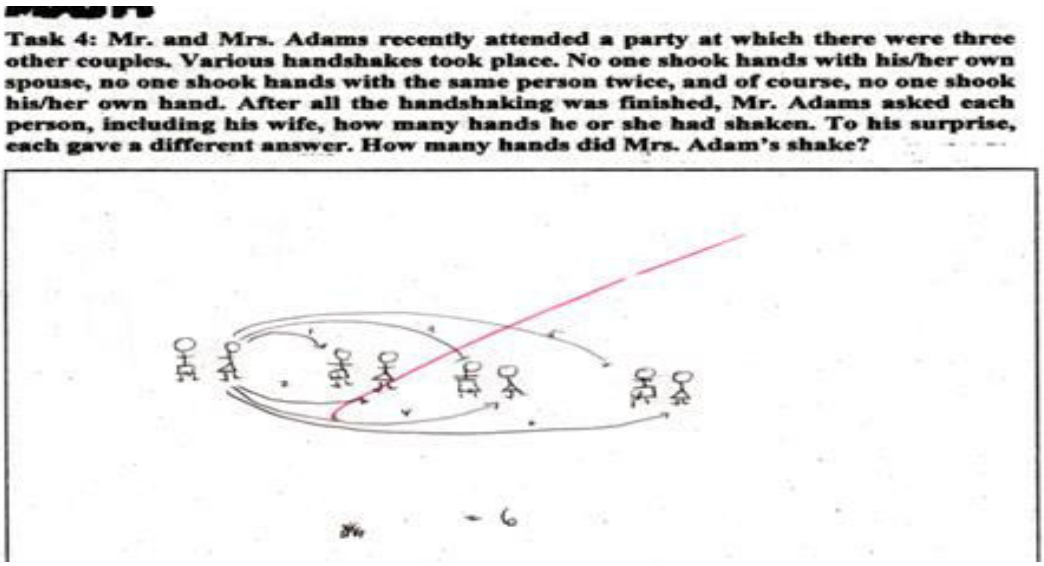
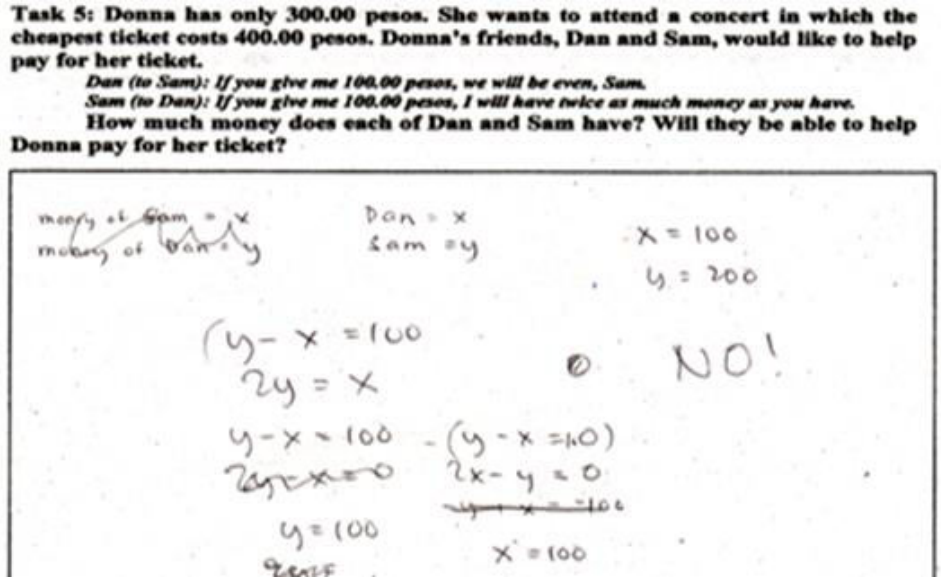


Figure 4. Use a Representation

The respondent used a visual depiction of the couple attending the party. But during her interview, she was not able to mention this strategy in solving.

10. Use of mathematical Expression or an Equation



ISSN: 2278-4012, Volume:7, Issue:3, July 2018

Figure 5. Use of mathematical Expression or an Equation

The student made use of mathematical expression or an equation but was not successful in doing so. He was not able to mathematically represent the unknowns in problem correctly. He was even self-evaluating. He knew that he was not on the right track in solving the problem; the proof of that metacognition was his note "NO" on his paper.

11. Use of the basic operations

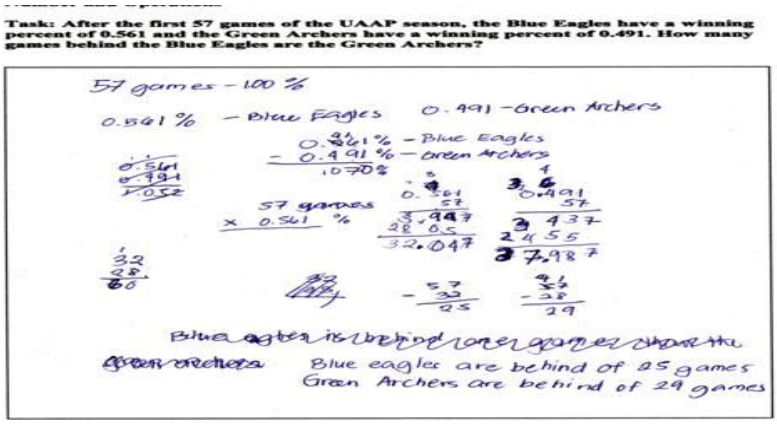
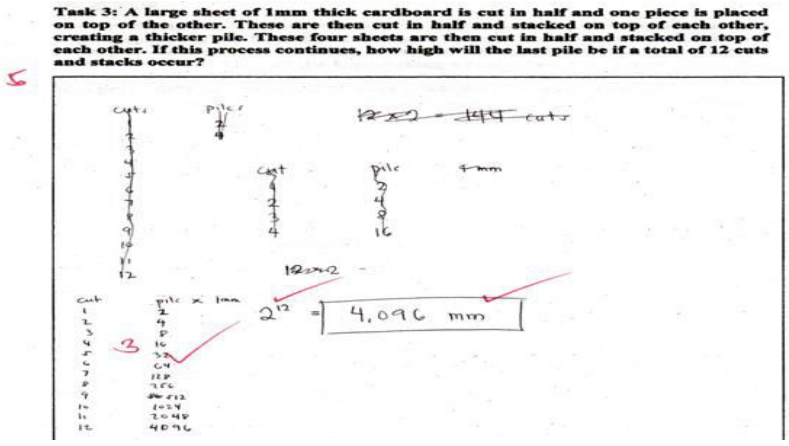


Figure 6. Use of the Basic Operations

The student made use of his skills in the four basic mathematical operations. He did not use a formula in solving nor a representation but all he did was manipulate the numbers using the fundamental operations. This was an example of the work of one of the poor problem solvers in the study.

12. Make a List, Find a Pattern and Derive a formula

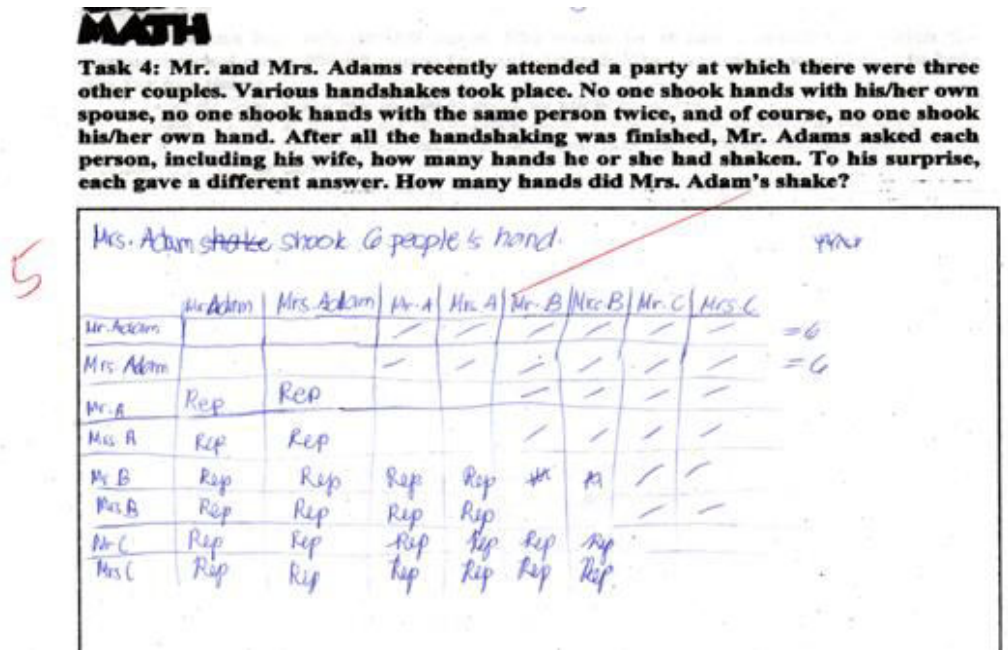


ISSN: 2278-4012, Volume:7, Issue:3, July 2018

Figure 7. Make a List, Find a Pattern and Derive a formula

Figure 7 is one of the outputs of a good problem solver among seniors. He combined the three strategies in solving mathematical problems namely; make a list, find a pattern and derive a formula. He stressed out that he is formula-dependent when solving a problem, but his works indicate otherwise. This was just one of the many proofs that students do know lots or few problem solving strategies but just don't know what they are. They had been using them unconsciously and were not able to recognize them by name.

13. Make a Table


MATH

Task 4: Mr. and Mrs. Adams recently attended a party at which there were three other couples. Various handshakes took place. No one shook hands with his/her own spouse, no one shook hands with the same person twice, and of course, no one shook his/her own hand. After all the handshaking was finished, Mr. Adams asked each person, including his wife, how many hands he or she had shaken. To his surprise, each gave a different answer. How many hands did Mrs. Adam's shake?

Mrs. Adam ~~shake~~ shook 6 people's hand.

	Mr. Adam	Mrs. Adam	Mr. A	Mrs. A	Mr. B	Mrs. B	Mr. C	Mrs. C	
Mr. Adam			/	/	/	/	/	/	=6
Mrs. Adam			/	/	/	/	/	/	=6
Mr. A	Rep	Rep			/	/	/	/	
Mrs. A	Rep	Rep			/	/	/	/	
Mr. B	Rep	Rep	Rep	Rep			/	/	
Mrs. B	Rep	Rep	Rep	Rep			/	/	
Mr. C	Rep	Rep	Rep	Rep	Rep	Rep			
Mrs. C	Rep	Rep	Rep	Rep	Rep	Rep			

Figure 8. Make a Table

Make a table is one of the many problem solving strategies that seniors used in solving mathematical problems. The example above illustrates the students' creativity in using this strategy in solving. Recall that one of the respondents earlier solved this problem using a representation and was right, too. A proof that there are many ways a problem can be solved correctly.

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

14. Guess and Test

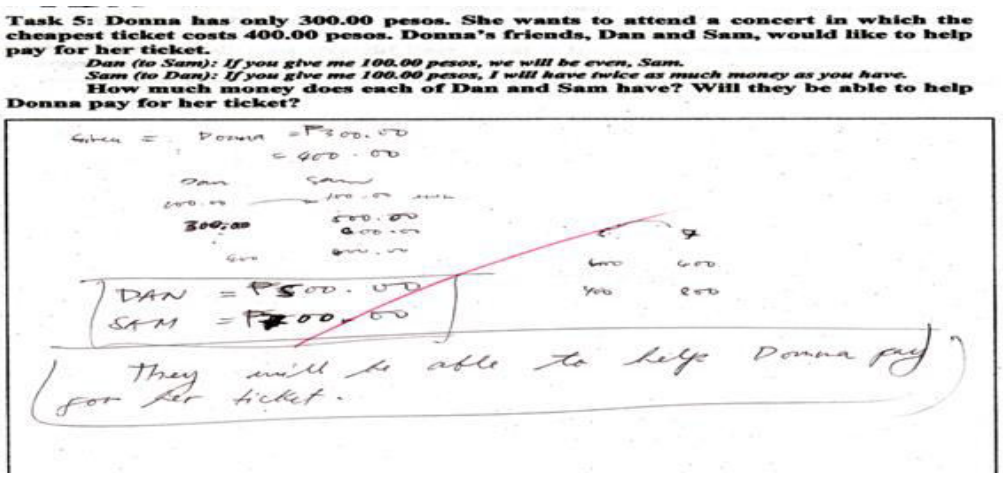


Figure 9. Guess and Test

This was one of the works of an average problem solver. During the interview, he claimed that he uses guessing or trial-and-error in unfamiliar problems. In this problem, he wrote down possible pairs of numbers (through *guessi*ng) and *checked* if they will meet the condition of the problem and then finally wrote down his answer.

15. Skip Counting and Adding Techniques

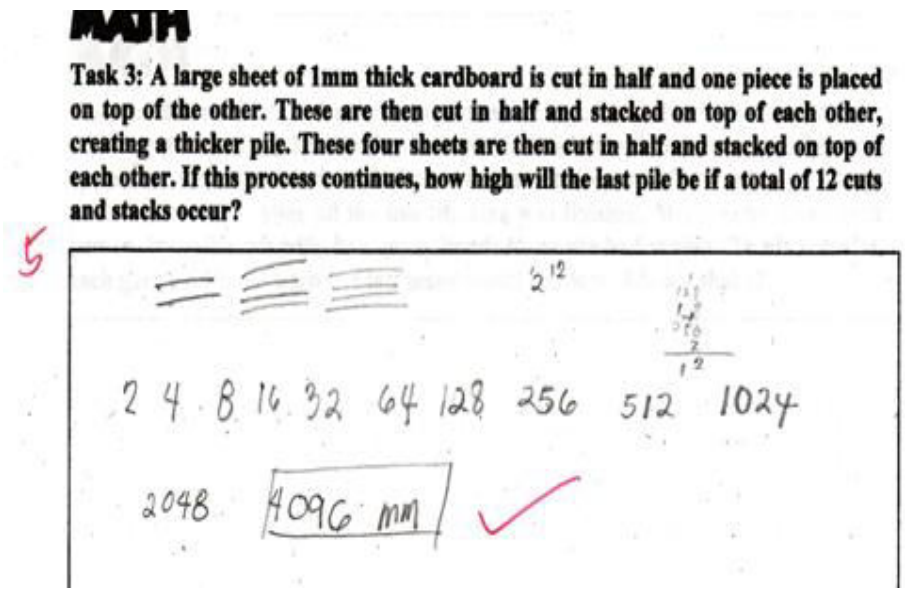


Figure 10. Skip Counting and Adding Techniques

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

In the preceding pages, the same problem was solved through the combination of make a list, find a pattern and derive a formula strategy. In here, the respondents solved the same problem using a different strategy: skip-counting and adding techniques. She also made use of illustrations prior to the actual skip counting technique. She even wrote the answer in the exponential form on her paper.

16. Use of Logic or Reasoning

Once upon a time, there was a girl named Pandora, who wanted a bright groom so she made up a few logic problems for the wannabe. This is one of them.
Based upon the inscriptions on the boxes (none or just one of them is true), choose one box where the wedding ring is hidden.

Golden box
The ring is in this box.

Silver box
The ring is not in this box.

Lead box
The ring is not in the golden box.

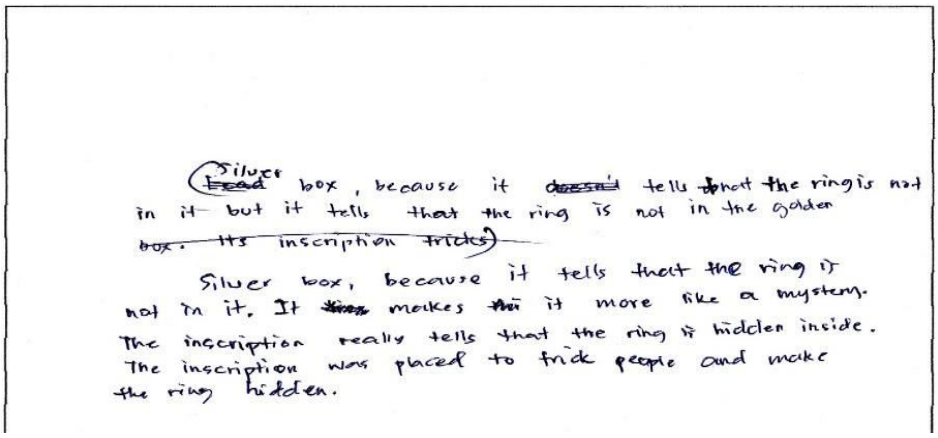


Figure 11. Use of Logic or Reasoning

As mentioned earlier, problem solving heuristics is problem-dependent. Some problems can be solved by many strategies (or ways) while others can be solved by a single strategy. Problems involving the use of logic can only be solved in a single way: of course using our common sense. And the problem above is one of the types of problem that can only be solved through the use of logic or common sense.

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

17. Draw a Figure and Use the basic Operation

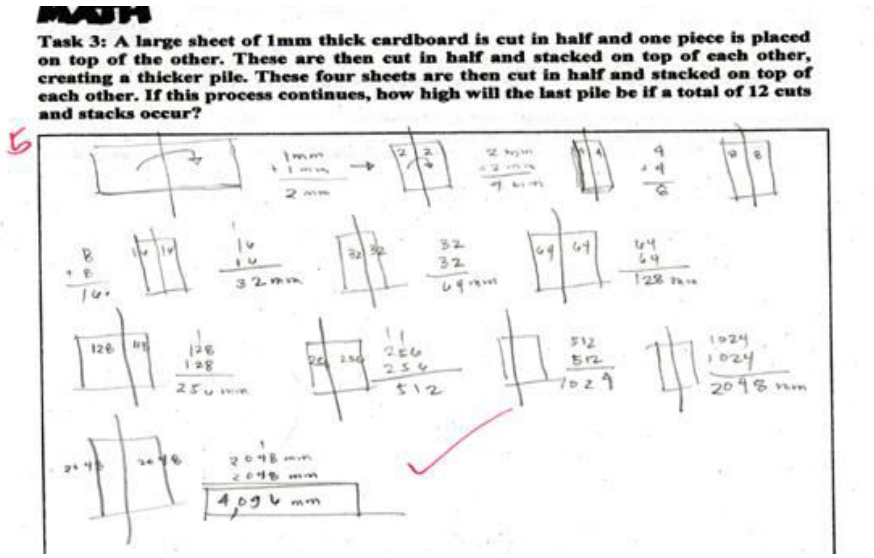


Figure 12. Draw a Figure and Use the Basic Operation

In the above output, the student made use of draw a figure technique in solving the problem. Others solved this problem using combinations of strategies while others just used a formula in solving this problem.

Most of the time, participants in this study did not know what a strategy is: they simply said that they solve math problems by reading/understanding them, visualizing/drawing, analyzing, and solving. If there is still time left, they usually checked their solution. The above results indicate that students had been using some of the problem solving heuristics but were unknowledgeable of its name *per se*.

Some respondents claimed that in solving mathematical problems, they usually skip difficult items and solve easy problems first. They do not stay long on difficult items in order not to waste their time (especially if exams are time-pressured) but solved them last when time warrants. Others find clues from the other items in the problem. Mostly, students were expecting examinations that would test their skills in a specific topic; hence, the formula in that topic will be used. They were so sure that teachers would never give problems that were never discussed in class. So, all they had to do is master the process presented by the teacher in solving a problem and familiarize the formula.

Garcia (2004), characterized solver's choice of solution strategy as in which she has augmented in the following versions:

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

Use of trial and error: The solver tries to guess the solution either wildly or with some attention paid to problem structure. This guessing technique may be carried out systematically or unsystematically.

Use of keywords: The solver tries to find the solution by using mathematical formulas that are chosen on the basis of the presence of particular keywords in the problem statement. Novotna defines three levels for the use of keywords: search in the memory (use of unrelated pieces of information existing in the long term memory), textation (use of partial relationships without a global understanding of the problem structure), and use of keywords with global understanding of the problem structure.

Use of Heuristics: Garcia (2004), describes *heuristics as* systematic search strategies for problem analysis and transformation. The use of heuristics does not guarantee that the solver will find the correct answer to a problem; however, because a heuristics induces a systematic planned approach –in contrast to the trial and error and the use of keywords strategies – it substantially increases the solvers probability of success in solving the problem (Garcia, 2004). The solver either has a global understanding of the problem structure and chooses an appropriate heuristics to solve the problem, or has no clear understanding of the problem structure but anyhow tries using heuristics, though not effectively. The use of a heuristics is considered effective when it yields the correct answer. It is ineffective when it is inappropriate or when there is a breakdown in the application of the heuristics process. Some of the examples of the heuristics are: pattern recognition, make a diagram, organized listing, logical deduction, work backward, use of algebraic expressions, divide and conquer, pursue parity, use of contradiction, and use of the pigeonhole principle.

CONCLUSIONS AND IMPLICATIONS

The problem solving heuristics used by the participants in this investigation are as follows: use of logic or common sense, look for a clue, use an equation, draw a picture, solve or recall similar or equivalent problem, make a list, find a pattern, and make a table, make a diagram, use of mathematical expressions, use a model and use a representation. Other solution strategies used by the participants in the study are: trial-and-error, guessing, guess-and-test, skip counting, use of the basic operations, use of adding techniques and recall and create formula. Participants in the study used the solution strategies as stand-alone or a combination of two or more strategies.

The findings of this study will inform educators to expose students to varied problems in mathematics so that they are able to develop the different solution strategies and problem solving heuristics. When teachers provide similar activities and problems to the students, their problem solving skills and strategies will just be limited to the ones presented and discussed to them.

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

Acknowledgement:

The author wishes to express its gratitude to the Central Mindanao University Administration and the Department of Science and Technology (DOST) – Science Education Institute (SEI) for the scholarship grant. This piece of work is a portion of the author's dissertation paper during her doctoral studies at De La Salle University (DLSU) – Manila.

References:

Allan, G. (2003). *A critique of using grounded theory as research method.* Electronic Journal of Business Research Methods, 2(1), 1-10. Retrieved July 10, 2009, from http://www.ejbrm.com/vol2/v2-il/issue1-art-allan.pdf.

Billstein, R., Libeskind, S., Lott, J. (2000). A problem solving approach to Mathematics for Elementary School Teacher (7th ed.). One Jacob Way, CA: Addison-Wesley.

Borkowski, J., Carr, M., & Pressely, M. (1987). "Spontaneous" strategy use: Perspectives from metacognitive theory. *Intelligence, 11,* 61-75.

Byrne, B. M. (1996). *Measuring self-concept across the life-span.* Washington, DC: American Psychological Association.

Carlson, M. & Bloom, I. (2005). *The Cyclic Nature of Problem Solving: An Emergent Problem Solving Framework.* Educational Studies in Mathematics, Vol. 58, No. 1 (2005), pp. 45-75.

Flavell, J.H.(1976). '*Metacognitive aspects of problem solving'*, in L.B. Resnick (ed.), The Nature of Intelligence, L.E.A. Hillsdale, NJ.

Fortunato, I. Hecht, D., Tittle, C.K., and Alvarez, L. (1991). *Metacognition and problem solving,* Arithmetic Teacher 39(4), 38-40.

Garcia, E.G. (2004). *Enhancing the problem solving Skills of the pre-service Mathematics teachers through a bridging course.* Unpublished Doctoral Dissertation. De La Salle, Manila.

Krulik, S., Rudnick, L. (1996). The new sourcebook for teaching reasoning and problem solving in junior and senior high school. Allyn and Bacon (eds.). Boston.

Limjap, A. & Candelaria, M. (2002). *Problem Solving Heuristics of College Freshmen: A Case Analysis.* The Asia-Pacific Education Researcher. December 2002, 11(2), 199-223.

ISSN: 2278-4012, Volume:7, Issue:3, July 2018

Martinez, M. (2008). What is problem solving. Retrieved November 19, 2008, from http://www.gse.uci.edu/person/mmartinez/documents/mmartinez_Problem_Solving.html

McLeod, D.B. (1994). *Research on affect and mathematics learning in the JRME: 1970 to the present*. Journal for Research in Mathematics Education, 24, 637-647.

Mevarech, Z.R. and Kapa, E. (1996). *The effects of a problem-solving based Logo environment on children's information processing components,* The British Journal of Educational Psychology. 66, 181-195.

Miles M.B., & Huberman, A.M. (1994). *Qualitative Data Analysis.* Sage Publication.

Montague, M. (2008). *Self-regulation strategies to improve mathematical problem solving for students with learning disabilities.* Learning Disability Quarterly. ProQuest Psychology Journal. 31, 37-44.

NCTM (1999). Principles and Standards for School Mathematics. Retrieved November 21, 2008, from http://standards.nctm.org/document/chapter7/data.htm

NCTM (2000). Principles and Standards for School Mathematics. Retrieved November 21, 2008, from http://standards.nctm.org/document/chapter7/data.htm

Pajares, M.F. (1992). *Teachers' beliefs and educational research: Cleaning up a messy construct*. Review of Educational Research. 62, 307 – 332.

Pajares, F., & Miller, D. M. (1994). *The role of self-efficacy and self-concept beliefs in mathematical problem-solving: A path analysis.* Journal of Educational Psychology, 86, 193-203.

Pandit, N.R. (1996). The creation of a theory. A recent application of the grounded theory method. The Qualitative Report, 2(4). Retrieved November 19, 2008, from http://www.nova.edu.sss/QR/QR2-4/pandit.html

Patton, M. Q. (1990). *Qualitative evaluation and research methods* (2nd ed.). Newbury Park, CA: Sage Publications.

Polya, G. (1945). *How to Solve It*, Garden City, N.Y.: Doubleday.

Strauss, A., & Corbin, J. (1990). *Basics of Qualitative research: Grounded theory procedures and techniques.* Newbury Park, CA: Sage.

Strauss, A., & Corbin, J. (1998). *Basics of Qualitative Research: Grounded theory procedures and techniques* (2nd ed.). London: Sage.

Yin, R.K. (1994). *Case Study research*. Sage Publications.