

STUDENTS' DIFFICULTIES IN SOLVING RIGHT TRIANGLE**MARY LOREEN V. CAYABYAB**

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Abstract: This study is quantitative and qualitative using case study method. An interview was done with some of the respondents from selected courses of the University of Northern Philippines, Vigan City Ilocos Sur during the school year 2016-2017. The purpose of this study is to determine the difficulties encountered by the students in solving right triangle problem. The difficulties were classified as comprehension, transformation and process skill. There were 154 student respondents. The respondents were found that the respondents have difficulty in comprehension. The factor is the belief system in Mathematics. It is recommended that the colleges should include activities related to Mathematics. Further, another study should be done to validate the result of this study.

Keywords: Right triangle, difficulty, comprehension, transformation, process skill

INTRODUCTION

Trigonometry is a branch of Mathematics that studies relationships between the sides and angles of a triangle. It is designed for students majoring in Mathematics, Science, Engineering or related fields. The content includes the study of trigonometric functions and applications. Trigonometry may not give us a direct application, but it is used in several ways in which we are enjoying today. Architects use trigonometry to calculate the structural load. The proper trajectory could be estimated, and the reason for the causes of collision in a car accident can be avoided.

The basic concepts of Trigonometry are on the relation between the sides of angles of the triangles.

It is one of the branches of mathematics which most of the students hate and struggle with. In the study of Mensah (2017), it was found out that the students' error in solving trigonometry problems was due to their weaknesses in basic arithmetical operations. This is the same as Orhun

(2015) who found out that the students did not develop the concepts of trigonometry certainly and that they made some mistakes.

Trigonometry is an application of Calculus and many other higher Mathematics. Problem solving is the ability to identify and solve problems by applying appropriate skills systematically, and it is a process- an ongoing activity in which we take what we know to discover what we do not know. As defined by Demir (2012), Trigonometry is one of the important topics in secondary school Mathematics requiring integration of different algebraic, geometric and graphical reasoning. This means it is crucial to learn trigonometry based on ratio definitions of the right triangle to be able to solve problems on the right triangle.

In education, the perception of mistakes is not since necessary it challenges conceptions of what defines success or failure. Mistakes are the basis for improvement. (Clifford, 2012). Therefore, it is of great importance to know the causes of difficulties so that the educators will be given an idea on how to address these difficulties and be able to help on how the students will be prevented from repeating the same mistakes resulting from their difficulties.

The role of Mathematics teachers is not just to record the scores of the students during exams. The inability to do something to correct these mistakes must be frustrating on the part of the students. Mistakes are signs to the students so that they will know where to focus and where to deliberate practices. Educators must admit that the scores reflected by these mistakes make the students feel stupid. It is for this reason that educators should change their perspectives on mistakes.

Some students do not notice what is causing the difficulties. It is not only about having other students give the correct answer but also about having them explain the solution. Some specific error patterns can cue an educator that a student not only uses an ineffective procedure to compute a problem, but that student also does not accurately understand an important Mathematics concepts which may lead to difficulties.

It is in this context that the researcher was motivated to analyze the difficulties of the students in solving Mathematics problems. This study will benefit Mathematical educators and learners. The findings of this study will help Mathematics teachers to look at the difficulties in more positive ways. For the students, they will be given an idea on their difficulties that they encounter thus making them aware so that the next time they encounter this in Math problems,

they will know how to overcome these difficulties. The result of this study will be of help of how the students could perform better in higher Mathematics.

Purpose of the Study

The purpose of this study was to analyze the students' difficulty in solving the right triangle. It is also the objective of this study to determine the causes of the students' difficulties in their attempt to solve right triangle problem.

Specifically, it sought to answer the following questions: 1.) what the nature of the difficulties encountered by the students in solving the right triangle problems? 2.) What are the factors contributing to these difficulties as perceived by the respondents' mathematics instructors? 3. In what step do the students have experienced difficulty in solving the right triangle?

Method

Research Design

To be able to come up with the research objectives, right triangle problem solving test and in depth semi structured interviews were chosen as the means to collect the data and all other necessary information. The qualitative-based research method was employed with some additional results from quantitative data analysis about the students' difficulties in solving the right triangle problem. Students' difficulties in solving the right triangle problem were analyzed from the interviews, which the difficulties were classified. Interview protocols were employed to identify the factors contributing to the difficulties and students' experience in solving right triangle problem.

This study made use of the case study method. The researcher employed a standardized, open-ended interview. Interviews are particularly useful for getting the story behind a participant's experience (McNamara, 1999).

Participants and Research Site

The study was conducted at the University of Northern Philippines, Vigan City Ilocos Sur during the school year 2016-2017. A total of 234 respondents were taken. The distribution of respondents could be seen from the table below.

Table I
Distribution of Respondents

Courses	Frequency
BS Architecture 2	16
BEeD III A	51
BEeD III C	42
BSIT II	35
BSED II	20
TOTAL	154

Instrumentation and Data Collection

For this study, a mathematical word problem in the right triangle was taken from the Plane Trigonometry course book by Pedro, et.al. (2009). The difficulty index of the problem was tested, and a $p=0.602$ was obtained which means that the item is at an average level. The problem deals with the application of the right triangle. By the definition of the mathematical problem, this is a problem that is amenable to being represented, analyzed and possibly solved, with the methods of mathematics. This can be a real-world problem with mathematical ideas that the students should know and could apply to the world to connect to other mathematical content and process to increase their knowing. The problem presented should at least one solution that can be understood by the students.

Permission was obtained from the respondents to audio tape the interview. The researcher assured the respondents that the data would be kept confidential and it will only be used for the study. The copy of the problem was distributed to the respondents. The instruction was explained to them that they have to write their complete solution starting with the given, the unknown, illustration, formula to be used, complete solution and the final answer.

The Problem

A student standing 130 meters from the base of a flagpole in the school open field observes that the angle of elevation of the top of the pole is $15^{\circ}23'$. Find the height of the flagpole.

Analysis of Data

The researcher used frequency count and percentage to get the descriptive findings of the difficulties of the student

The researcher analyzed the students' exam papers to see the difficulty patterns that the students made in their right triangle problem then compared the patterns across tried to see the consistency of occurrence. Through these difficulty patterns and with the analysis of the interview, the researcher tried to detect students nature of the difficulty.

In the same way, interview protocols were analyzed and prepared transcripts to see the themes which have a similarity with the Newman Error Analysis procedures. The transcripts were then grouped into comprehension difficulty, transformation difficulty, and process skills difficulty.

RESULTS AND DISCUSSIONS

Table 2 presents the frequency distribution of the type of answers of the respondents.

Table 2
Frequency Distribution of the Type
Answer of the Respondents

COURSES	TYPE OF ANSWER			
	Correct		Wrong	
	f	%	f	%
BS Architecture II	9	5.84	7	4.55
BEeD III A	6	3.90	35	22.73
BEeD IIIC	7	4.55	35	22.73
BSIT II	10	6.49	25	16.23
BSED II	18	11.69	2	1.29
As a whole	50	32.47	104	67.53

It could be reflected from the table that only 50 or 32.47% of the respondents were able to get the answer correctly and more than half (67.53%) got the wrong answer. This is evidence that the respondents have difficulties in solving a problem in the right triangle since more than half of the respondents did not answer correctly.

It could also be seen from table 3 the frequency of the nature of the difficulties of the respondents. The nature of difficulties identified is: comprehension, transformation, and process skills. Comprehension difficulty occurs when a student does not know how to approach a given

trigonometric problem from the concepts given. Transformation difficulty occurred when a student had difficulty in the computation process. Process skills difficulty occurred when a student failed to translate and manipulate trigonometric ratios.

Table 3
Frequency Distribution of Nature
Of Difficulties of the Respondents

COURSES	NATURE OF DIFFICULTIES							
	Comprehension		Transformation		Process Skills		TOTAL	
	f	%	f	%	f	%	f	%
BS Architecture II	5	4.81	2	1.92	0	0.00	7	6.73
BEeD IIIA	21	20.19	4	3.85	10	9.62	35	33.65
BEeD IIIc	18	17.31	16	15.39	1	0.96	35	33.65
BSIT II	24	23.07	1	0.96	0	0.00	25	24.04
BSED II	0	0.00	2	1.92	0	0.00	2	1.93
As a whole	68	65.38	25	24.04	11	10.58	104	100.00

Difficulty in Comprehension

The first step for the student to do is to identify the given, which are: 130 meters – the distance of the student from the base of the flagpole and $15^{\circ}23'$ - angle of elevation from the student to the top of the pole.

Most of the respondents were able to identify the given but not able to decode the problem properly in the illustration maybe because they were confused on how to identify the given data. Most of them find easy to illustrate the problem but not in identifying the parts properly. Illustration of the problem is important as it will reflect the given data in the problem to be able to identify the unknown. It is in this part the respondents failed to reflect the parts of the illustration correctly. Some of the interviews in this scenario is shown below.

Student 1

MLV: What are the given?

Student 1: (no answer)

Student 2

MLV: What are the given in the problem?

Student 2: Yung magagamit ko Mam?

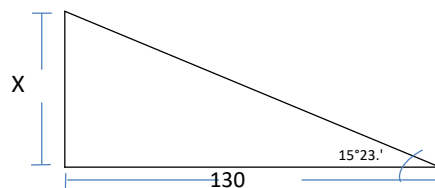
MLV: The data that are necessary and stated in the problem

Student 2: Sine?

Gur (2009), stated the students have some misconceptions and obstacles about trigonometry. One of the two obstacles to effective learning was that trigonometry and other concepts related to it were abstract and non-intuitive. In the same way, the finding of this study

agrees with Clement (2008) who said that knowing how to decode a word problem is only one facet of being the successful problem solver.

The second step in comprehension is for the respondents to illustrate the situation defined in the problem and to properly label the parts of the illustration as stated in the problem. The proper illustration of the problem is shown below:



It was found out in this step that the respondents could illustrate where the student is standing from the flagpole, and almost all of them was able to label properly the distance of the student from the flagpole. However, it cannot be denied that most of them maybe had forgotten or unfamiliar with the definition of the angle of elevation because they failed to identify this in their illustration.

Illustrations of some of the respondents are shown below.

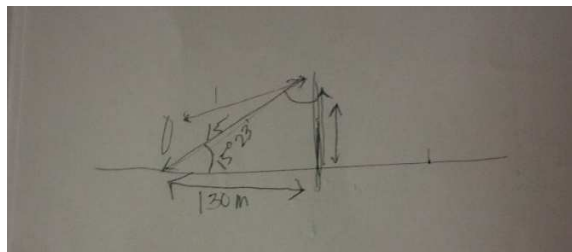


Illustration 1: First sample illustration of respondent

In this illustration, the respondent is not sure which part of the illustration the angle of elevation should be placed. The first location is correct, but then the respondent wrote 15 on the hypotenuse which did not lead him to the next step.

The second illustration shows that the angle of elevation was not identified correctly. This proves that the respondent had forgotten the definition of angle of elevation.

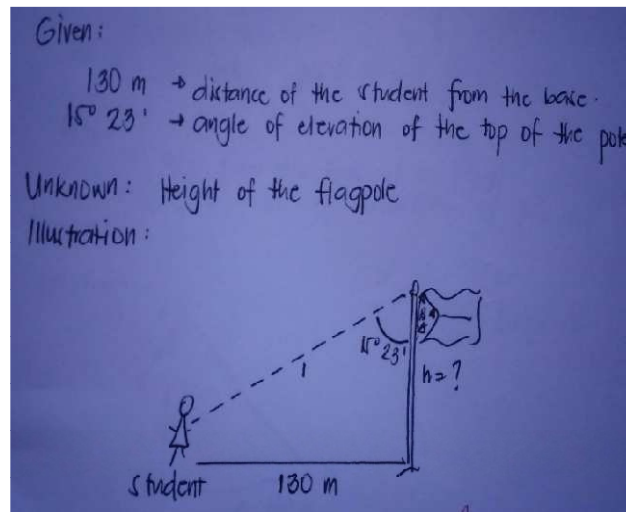


Illustration 2: Second sample illustration of the respondents

The third illustration shows the part of the triangle which is the hypotenuse and was labelled as the angle of elevation with a value of $15^\circ 23'$.

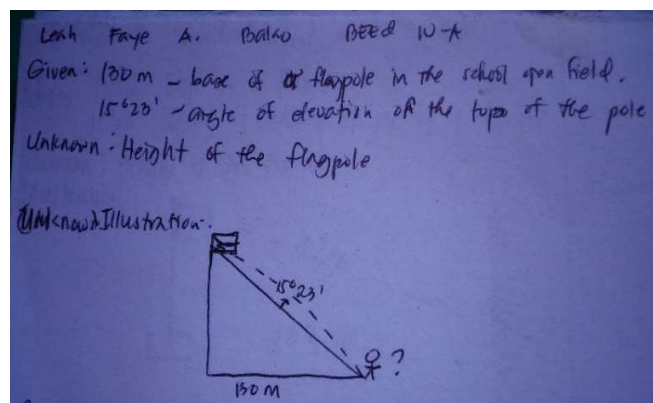


Illustration 3: Third sample illustration of the respondents

The respondent labelled the hypotenuse as the angle of elevation that may lead him to inability to go beyond this stage. Also, based on the interview done, some of the respondents are unfamiliar with what angle of elevation is. This was confirmed based from the interview.

Student 3

MLV: where is the angle of elevation stated from the problem?

Student 3: Eto (pointing at the angle formed by two perpendicular lines)

MLV: Wait lang ha, the angle of elevation is the angle formed from the eyesight of the observer to an object at a higher level di ba?

Student 3: (No answer)

Student 4

MLV: So do you think this is the angle of elevation?

Student 4: ah...

MLV: Where is it?

Student 4: Here? (pointing at the wrong angle)

MLV: That is formed by two perpendicular lines. What kind of angle is that?

Student 4: ah so eto ba un mam?

MLV: From the boy.

Student 4: ah angle (pause) so eto ba mam?

MLV: ok.

This is the same with the research findings of Haghverdi (2012) revealed that students' errors when solving arithmetic word problem results from the lack of structural knowledge when solving the geometric word problems, the lack of semantic, intuitional and structural knowledge were the causes of the students' errors.

The fourth illustration displays that none of the given were correctly identified by the respondent.

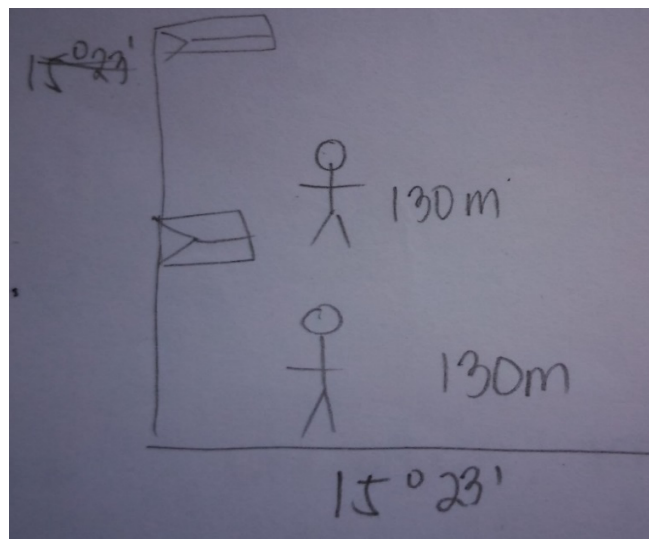


Illustration 4: Fourth sample illustration of the respondents

Some of the interviews of students who had this difficulty can be seen below.

MLV: What is the unknown?

Student 5: Height.

MLV: What is given?

Student 5: The distance and the angle of elevation

MLV: Please illustrate.

Student 5: (Can not illustrate the problem, does not know where the flagpole is, the student and other given in the problem.)

MLV: Illustrate.

Student 6: (Looking at the problem and paper)

MLV: Where is the flagpole? Where is the student?

Student 6: (no answer)

MLV: Where is the student from your illustration?

Student 6: (looking at the paper)

MLV: Ok...What do we mean by the "130 meters"?

Student 6: The distance.... Here?

Difficulty in Transformation

The next stage is for the respondents to identify the formula to be used to solve the problem. Since the given are an angle of elevation and the adjacent side of a right triangle as seen from the illustration, the trigonometric function to be used to solve for the height of the flagpole is the tangent of an angle. The proper formula to be used is

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{x}{130} \text{ where } x \text{ is the unknown side.}$$

The table shows that there are 25 or 24.04% of the respondents who had difficulty in transformation. Some of the respondents had a hard time to define what to do next at this stage. They tried to recall steps from past lessons but were not confident on the correct approach on how to solve the problem simply because they are not familiar with the trigonometric functions needed in solving of a problem.

$$\begin{aligned} \cos &= \frac{\text{adj}}{\text{hyp}} \\ \cos 15^{\circ} 23' &= \frac{130 \text{ m}}{\text{hyp}} \\ \text{hyp} &= \frac{130 \text{ m}}{\cos 15^{\circ} 23'} \\ &= 134.8304295 \text{ m} \\ &= 134.83 \text{ m} \text{ height} \end{aligned}$$

Illustration 5: First sample on Transformation difficulty

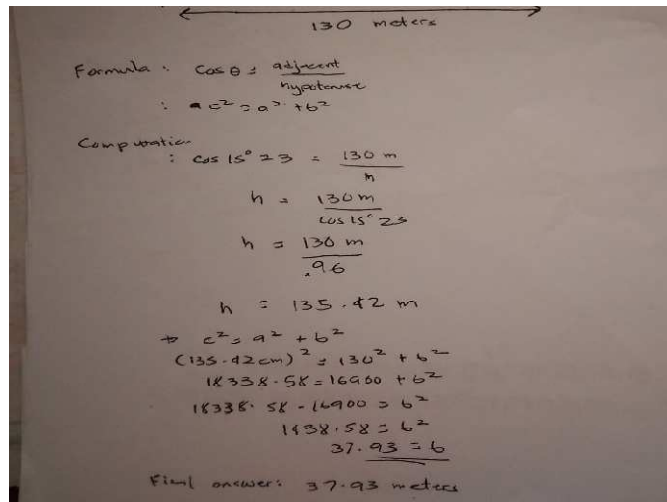


Illustration 6: Second sample on Transformation difficulty

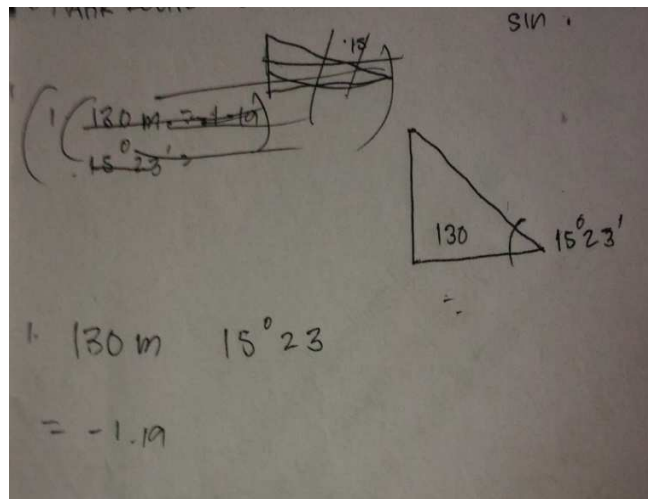


Illustration 7: Third sample on Transformation difficulty

Gooding (2009) mentioned in the article that there is a difficulty of the students in recognizing and imagining the context in which a word problem is set, or their approach is

altered by the context in which the word problem is given. This is similar to some of the students based on their interview.

Student 7

MLV: Ok. What formula are you going to use?

Student 7: Angle?

MLV: What is to be found?

Student 7: Here (pointing at the correct part of the illustration showing the unknown).

MLV: Ok. So what is the formula to be used?

Student 7: Cosine?

Student 8

MLV: What formula are you suppose to use?

Student 8: Sine

MLV: Why sine?

The same thing was stated by Trance (2013) who found out that 70% of the errors were comprehension and transformation errors.

Similarly, the findings of Fuifui, et. al. (2013), states that some of the respondents of the study were able to solve questions that tested on procedural skills. However, they did not perform due to the lack in the conceptual aspects of trigonometry. They do not understand the properties or principles needed to solve the problem. Other observations include lack of knowledge on trigonometric functions, failing to understand the question which is due to inadequate basic knowledge of trigonometry and fail to recall formula.

Difficulty in Process Skills

The final stage is on the computation of the unknown. After the correct formula was identified, the final step is to compute for the unknown. This will be done properly if the respondent has a good manipulative skills. The correct solution is shown below.

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan 15^{\circ}30' = \frac{x}{130}$$

$$(\tan 15^{\circ}30')(130) = x$$

$$(0.275)(130) = 35.77 \text{ meters}$$

The table reflects that there are 11 or 10.58% of the respondents who had no difficulties in comprehension and transformation but the difficulty in the process skill. From the solutions of the respondents, they were able to move on to the last phase but was not able to arrive at the correct answer. There are respondents who do not know the steps involved in solving the

problem. They were allowed to use the scientific calculator to solve for their final answer. The wrong answer that they had obtained could be seen from the illustrations.

Abdullah (2015) has mentioned that the result of his research indicates that students have problems in interpreting Mathematical problem, failed to devise a strategy and develop a strategic plan which eventually led to the error in choosing the operations involved and failed to state the answers. This may also be the same to the respondents why they got the incorrect final answer because of wrong operations.

Some of the illustrations are shown.

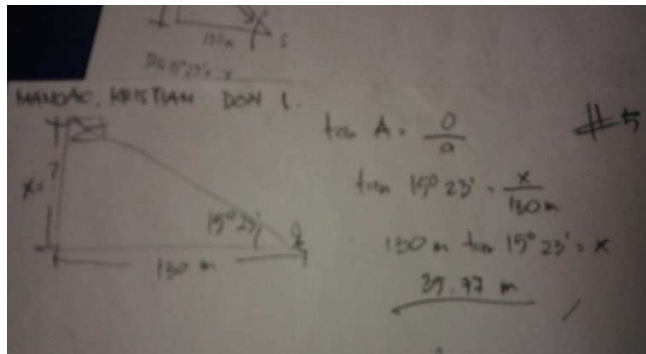


Illustration 8: First sample on process skill difficulty

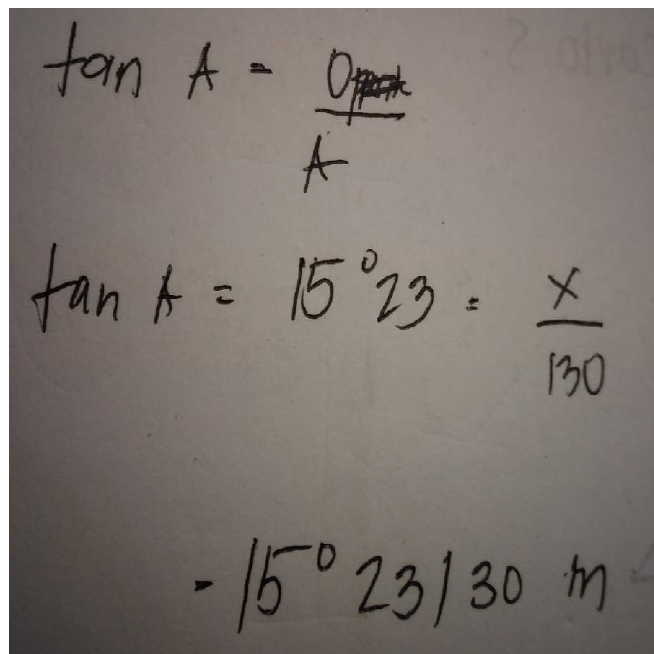


Illustration 9: Second sample on process skill difficulty

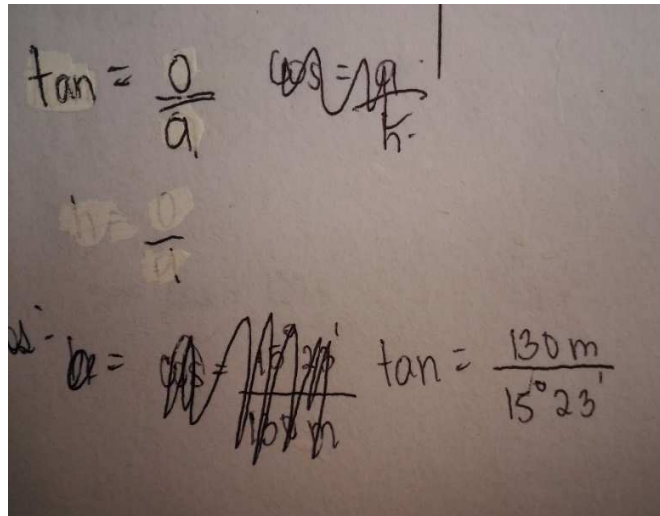
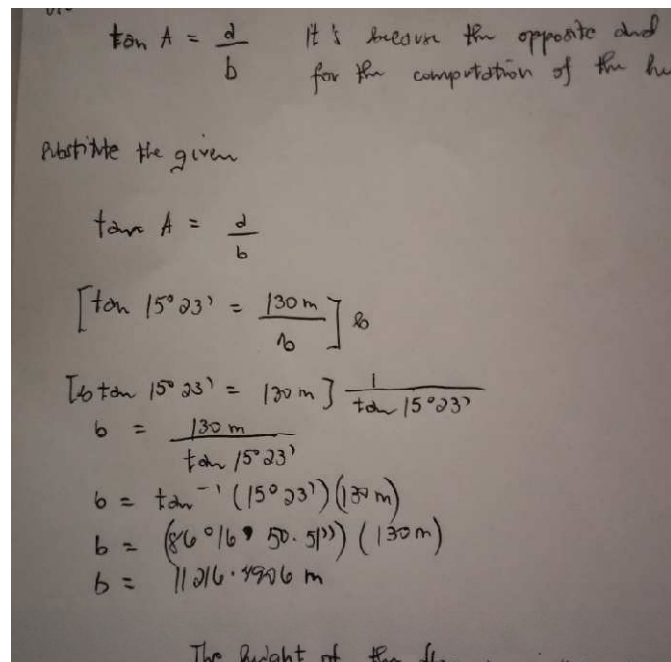


Illustration 10: Third sample on process skill difficulty

Illustration 11: Fourth sample on process skill difficulty
sample on process skill difficulty

The findings of this study agree with the findings of Fuifui et al. (2013), states that some of the respondents of the study were able to solve questions that tested on procedural skills. However, they did not perform due to the lack of the conceptual aspects of trigonometry. They do not understand the properties or principles needed to solve the problem. Other observations

include lack of knowledge on trigonometric functions, failing to understand the question which is due to inadequate basic knowledge of trigonometry and fail to recall formula.

Factors contributing to this difficulties as perceived by the respondents' instructors

The belief system in Mathematics. This is the attitude, disposition, and perception of the respondents in solving Mathematics problems. As one of the Instructors of the students, the researcher observed that the respondents are more interested to do designs of different models. This is a similar observation from another faculty who were also the instructors of the respondents. Two faculty members were interviewed to confirm this.

MLV: What do you think makes the students have the difficulty in problem solving? Not even in illustrating the problem.

Mam: The Architecture students are not mathematically inclined, and it was not the priority requirement of the course. They are poor in Mathematics, maybe just 5% of them can do Math.

This is the same observation with the other faculty who said the same thing.

MLV: What general observation can you make thus causing the students' attitude on their other subjects?

Sir: The students are more concentrated on other drawing subjects. They do not have the focus. They lost their momentum. They do not even take down notes. They are more engaged in designing.

Corte (2009), further said, the relevance of students' beliefs about the self in relation to mathematics, and more specifically of the conceptions of their competence in mathematics and their views on the personal relevance of mathematics. The clustering of these two subcategories in one factor indicates that students who are confident about their mathematical ability are mostly also the ones who are convinced about the relevance of mathematics; this can create a solid motivational basis.

Successful mathematical problem solving depends upon many factors and skills with different characteristics. In fact, one of the main difficulties in learning problem solving is the fact that many skills are needed for a learner to be an effective problem solver.

Steps the students experience difficulty in solving problems

From the interview done to the respondents, it was observed that along the steps done in the solution of the Mathematical word problem, the respondents have difficulty along comprehension. The respondents are hard up in making the correct illustration and in identifying the parts of the triangle. This is the same observation by the two Mathematics Instructors who are teaching the respondents in the same subject.

From the findings of Trance (2013), more than 70% of the errors found were comprehension and transformation errors. In the same way, from the article of International J Education Science, the findings of the study indicated that mathematical language appeared to influence learners' comprehension when solving mathematical word problems. Also, the study of Rodriguez (2012) reveals that students seemed to encounter issues in comprehending word problems statement which resulted in ineffective teaching and learning practice.

Most of the students who were able to proceed to the next step had difficulty on the strategy. The students do not know ways to approach and solve the problem. This is the same in the study of Tambychik (2010). It was stated that the data findings showed that respondents lacked in mathematical skills such as number fact, visual-spatial and information skills. Information skills were the most critical. The deficiency of these Math skills and also of cognitive abilities in learning inhibits the mathematical problem solving.

Those students who were able to determine the formula but had difficulty in the process skill do not know how to proceed with the computation. Some are not familiar on how to apply the trigonometric formula. The students lack the necessary definition to continue solving the problem. Although they know the trigonometric function to be used, they are not familiar with the parts of the triangle to use concerning the given angle. Thus preventing them to go to the next step.

MLV: Have you ever experienced their correct formula and yet wrong final answer?

Mam F: Yes, most of the time. I would say this is because they are not even familiar with the calculator that they are using. They do not spend the time to know the different short cuts

MLV: Have you ever experienced of their correct formula and yet wrong final answer?

Sir M: They have the correct formula but they do not know how to manipulate the data. There are those who had manipulated correctly the data and had executed the computation but copied the wrong answer. They do not even review their final answer.

MLV: Have you ever experienced their correct formula and yet wrong final answer?

Sir J: Yes for most of the students. I think they get too excited to get rid of what they are solving that they do not review their final answer.

On the other hand, there were respondents who were able to proceed to the next step but had given the wrong final answer. Again, this is the same observation with the other faculty handling the respondents in other natural sciences class like Physics and Statistics.

Conclusions and Recommendations

Based on the results of this study. The difficulties of the respondents are comprehension, transformation and process skills. The factor contributing to this difficulty is the belief system of the respondents in Mathematics. The steps used in the thought processing of the subjects as they engage in problem during the interview are the following: comprehension, transformation, and process skill.

Based on the conclusions, the following are recommended: 1.) The mathematics teacher should familiarize students in using trigonometric terms like angles, the angle of elevation and sides of a triangle. 2.) The mathematics teacher who is teaching Plane Trigonometry should spend lesser time on preliminary topics and give more time on the areas needed by the students to help them solve applications to right triangle easier like in determining sides and angles of a right triangle. 3.) The colleges should include activities during the celebration of their college week that will motivate students to learn and love Plane Trigonometry so that their attitude and disposition towards Plane Trigonometry will change. 4.) The colleges should sponsor the workshop to the Mathematics teachers on how to incorporate and improve comprehension, transformation, and process skill. Future study should be done using more problem solving problems on right triangle to validate the findings of this study.

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